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# PROJECT RAND

## STRUCTURAL WEIGHT ANALYSIS WING WEIGHT EQUATIONS

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## SUMMARY

This report presents some refinements to the wing weight estimation methods outlined in RAND Report R-100.<sup>(1)</sup> Since Report R-100 was published, the weight equations have been applied to a number of modern airplanes to determine certain empirical constants and to observe the relative size of the various terms in the equations. Some of these terms were found to be relatively small. Therefore, it was possible to shorten the final equations with little loss of accuracy.

This recent study also showed that the original assumptions regarding the distribution of dead weight in the wing needed revision. A more accurate method of accounting for the effects of dead-weight loads is outlined in this report.

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For references, see p.39.

## SYMBOLS

- $A$  = area
- $b$  = wing span (aerodynamic), in.
- $b_s$  = wing span (structural) ( $= b/\cos \Lambda$ ), in.
- $C_c$  = intercept value for  $\bar{t}/L_0$  for type of construction used in flanges of the structural box
- $C_r$  = intercept value for  $\bar{t}/h$  for type of construction used in ribs
- $C_s$  = intercept value for  $\bar{t}/h$  for type of construction used in the shear material of the structural box
- $\bar{C}$  = mean aerodynamic chord, in.
- $\bar{C}'_m$  = integrated effective moment coefficient
- $\bar{C}_s$  = structural mean aerodynamic chord ( $= \bar{C}/\cos \Lambda$ ), in.
- $d$  = ratio of thickness taper ratio to planform taper ratio ( $= m/\lambda$ )
- $F_a$  = equivalent axial allowable stress for bending, lb/sq in.
- $F_{ar}$  = allowable compressive stress in rib flanges, lb/sq in.
- $h$  = maximum wing depth at any span-wise station, in.
- $J_{LT}$  = weight of leading and trailing edge structure (including flaps and ailerons) divided by the total wing area
- $J_n$  = inertia-load correction factor (as used in Report R-100)
- $J_{nb}$  = inertia-load correction factor for bending loads
- $J_{ns}$  = inertia-load correction factor for shear loads
- $J_s$  = shear reduction factor (as used in Report R-100)
- $J'_s$  = integrated shear reduction factor
- $K_B$  = width of wing structural box expressed as a fraction of the chord
- $K_L$  = rib spacing expressed as a fraction of chord ( $= L/C$ )
- $K_h$  = maximum wing thickness, expressed as a fraction of chord
- $k_L$  = ratio of effective column length to actual length ( $= L_0/L$ )

$k_b$  = span-distribution factor (as used in Report R-100)  
 $k_{ba}$  = span-distribution factor for airloads (also equal to  $k_{i,sa}$ )  
 $k_{bd}$  = span-distribution factor for inertia loads (also equal to  $k_{i,sd}$ )  
 $k_e$  = wing effective depth factor  
 $k_{ib}$  = bending integration factor (as used in Report R-100)  
 $k_{iba}$  = bending integration factor for airloads  
 $k_{ibd}$  = bending integration factor for inertia loads  
 $k_{is}$  = shear integration factor (as used in Report R-100)  
 $k_{isa}$  = shear integration factor for airloads  
 $k_{isd}$  = shear integration factor for inertia loads  
 $k_{rf}$  = ratio of rear spar depth to airfoil maximum depth ( $= h_{rf}/h$ )  
 $k_s$  = ratio of average spar depth to airfoil maximum depth ( $= h_{ave}/h$ )  
 $k_x$  = nonoptimum factor for wing primary structure  
 $k'_x$  = nonoptimum factor which includes primary and secondary structure  
 $M$  = bending moment, in.-lb  
 $M_a$  = bending moment due to airloads, in.-lb  
 $M_d$  = bending moment due to inertia loads, in.-lb  
 $m$  = thickness taper ratio ( $= h_T/h_R$ )  
 $m_{ar}$  = ratio of allowable rib flange stress to equivalent axial stress ( $= F_{ar}/F_a$ )  
 $m_s$  = ratio of effective shear stress to equivalent axial stress ( $= F_{s_0}/F_a$ )  
 $n_f$  = ultimate load factor  
 $P$  = load, lb  
 $q_v$  = dynamic pressure, lb/sq ft  
 $S$  = wing area, sq ft  
 $\bar{t}_{r0}$  = minimum average thickness for ribs, in.  
 $V_a$  = shear due to airloads, lb  
 $V_d$  = shear due to inertia loads, lb  
 $V_R$  = value of shear measured at the root section, lb  
 $W_g$  = airplane gross weight, lb  
 $W_w$  = weight of wing, lb

$W_{wc}$  = weight of wing and wing contents, lb

$\eta$  = nondimensional span-wise distance measured from the  
root section

$\lambda$  = planform taper ratio ( $= C_T/C_R$ )



## I. INTRODUCTION

RAND Report R-100 outlined procedures for applying optimum design methods to structural weight analysis of the wing. Subsequent to the publication of Report R-100, the wing weight equations were applied to a number of modern airplanes to check the values of calculated weight against the actual weight. This preliminary study showed the need for revision of some of the original simplifying assumptions regarding the distribution of dead weight within the wing.

The procedure for estimating wing weights may generally be analyzed in two parts:

1. The estimation of the loads applied to the structure, based on a knowledge of the geometric properties of the wing and of certain loading parameters
2. The estimation of the weight of structure necessary to carry these loads.

The accuracy with which weights can be estimated depends directly on the accuracy with which the designing loads are estimated. The revision of the integration factors represents a means of improving the results of the weight equations by use of a more accurate method of determining the wing design loads.

For Report R-100, the airloads on the wing structure were determined by use of Weissinger's method.<sup>(2)</sup> These loads in turn were used to calculate integration factors as functions of planform taper ratio, sweepback, and thickness taper ratio. The integration factors represent, in nondimensional form, the volume of material necessary to carry the airload shear or bending moment for a given wing configuration. However, to obtain the net shear and bending moments, the inertia loads must be taken into account. This may be done in either of two ways: (1) by subtracting inertia loads from the airloads and working with net loads only, or (2) by calculating integration factors separately for airloads and for inertia loads and using these factors to calculate the net result. The second of these two methods is used here because of its greater convenience.

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For references, see p. 39.

Another revision made in the original equations is a change in the terms representing rib weight. These changes have been included in order to make the equations more accurate. Derivation of the new terms is given in Appendix C.

When the original equations of Report R-100 were tried out on wings corresponding to the configurations and loadings of several modern wings, it was found that some of the terms could be neglected with no appreciable error. Therefore it was possible to write the wing weight equation in a shortened, more convenient form. This shortened form of the equation is given in Section III.

## II. CHANGES IN THE ORIGINAL WING WEIGHT EQUATIONS

The expression for the volume of the optimum wing structural box material is given in Report R-100 as

$$\text{Vol} = \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + C_r \frac{K_h^2 K_B}{K_L} \right) 144 \bar{C} S + 2 \frac{K_B \bar{C}' q_v}{k_{rf} K_h F_y} \bar{C} S$$

$$+ \frac{1}{2} \frac{J_n n_f W_g}{F_a} \left\{ \left[ \frac{1}{2} \frac{k_{is} J_s}{m_s} + \frac{1}{3} k_{ib} \frac{k_b}{k_e} \frac{b/2}{h_R} \right] b + \frac{K_B}{m_s} \bar{C} \right\} \quad (1)$$

(See Report R-100, Eq.89.)

This equation comprises the following expressions for the volume of the different structural components:

### Shear Material

$$\text{Vol} = 2C_s k_s^2 K_h^2 (144 \bar{C} S) \quad \text{size term} \quad (1a)$$

$$+ \frac{1}{2} \frac{J_n n_f W_g b}{F_a} \left( \frac{1}{2} k_{is} \frac{J_s}{m_s} \right) \quad \text{loading term} \quad (1b)$$

### Bending Material

$$\text{Vol} = C_c k_L K_L K_B (144 \bar{C} S) \quad \text{size term} \quad (1c)$$

$$+ \frac{1}{2} \frac{J_n n_f W_g b}{F_a} \left( \frac{1}{3} k_{ib} \frac{k_b}{k_e} \frac{b/2}{h_R} \right) \quad \text{loading term} \quad (1d)$$

### Rib Material

$$\text{Vol} = C_r \frac{K_h^2 K_B}{K_L} (144 \bar{C} S) \quad \text{rib shear material (size)} \quad (1e)$$

$$+ \frac{1}{2} \frac{J_n n_f W_g}{F_a} \left( \frac{K_B \bar{C}}{m_s} \right) \quad \text{rib shear material (loading)} \quad (1f)$$

$$+ 2 \frac{K_B \bar{C}' q_v}{k_{rf} K_h F_y} \bar{C} S \quad \text{rib flange material} \quad (1g)$$

One part of Eq.(1) found to need revision is the expression for the volume of rib material. This change is made so that the equation describes more accurately the present practices in rib construction.

In order to provide a minimum gauge restriction on the rib shear material rather than the minimum proportion restriction provided by Eq.(1), the volume of the size term for rib shear material is rewritten as

$$Vol = \frac{\bar{t}_{r0} K_B K_h S(144)}{K_L} \quad (2)$$

where  $\bar{t}_{r0}$  = minimum average thickness of rib shear material. The derivations for the new rib-material terms are presented in Appendix C.

The expression for the volume of rib flange material, given by Eq.(1g), is determined from a consideration of the amount of flange material necessary to resist chordwise bending moments from the trailing edge structure. Since some of the parameters in Eq.(1g) may be difficult to evaluate in preliminary design stages, the volume of rib flange material will be based on a different set of parameters.

The revised expression for rib flange material presented in Eq.(3) gives the amount of material necessary to carry the axial loads associated with chordwise bending moments. In this case, the bending moments result from the transfer of normal airloads from the point of application in a chordwise direction to the shear resisting material in the spars. The expression for the volume of flange material is

$$Vol = \frac{J_n W_g n_f K_B^2 \bar{C}}{4 K_h F_{ar}} \quad (3)$$

where  $F_{ar}$  = allowable axial stress in the rib flanges. The derivation of Eq.(3) is given in Appendix C.

When Eqs.(2) and (3) are substituted for the corresponding rib terms (1e) and (1g) in Eq.(1), the volume of wing structural material becomes

$$Vol = \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_B K_h}{K_L \bar{C}} \right) 144 \bar{C} S$$

$$+ \frac{1}{2} \frac{J_n n_f W_g}{F_a} \left\{ \left[ \frac{1}{2} k_{is} \frac{J_s}{m_s} + \frac{1}{3} k_{ib} \frac{k_b}{k_e} \frac{(b/2)}{h_R} \right] b + K_B \bar{C} \left[ \frac{K_B}{2 K_h m_{ar}} + \frac{1}{m_s} \right] \right\} \quad (4)$$

where

$$m_{ar} = \frac{F_{ar}}{F_a}$$

The term (1f), representing the loading term for rib shear material, is not changed.

The optimum weight of the wing structural box is found by multiplying Eq.(4) by the density  $w$ . Using a single overall nonoptimum factor  $k_x$  and adding the weight of leading and trailing edges  $J_{LT}S$ , the total wing weight is written

$$W_w = wk_x \left\{ \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_B K_h}{K_L \bar{C}} \right) 144 \bar{C} S \right. \\ \left. + \frac{1}{2} \frac{J_n n_f W_g}{F_a} \left[ \left( \frac{k_{is}}{2} \frac{J_s}{m_s} + \frac{k_{ib}}{3} \frac{k_b}{k_e} \frac{(b/2)}{h_R} \right) b \right. \right. \\ \left. \left. + K_B \bar{C} \left( \frac{K_B}{2K_h m_{ar}} + \frac{1}{m_s} \right) \right] \right\} + J_{LT} S \quad (5)$$

where  $w$  = density of the material

$k_x = \frac{\text{actual weight of structural box}}{\text{optimum weight of structural box}}$

$J_{LT}S$  = weight of leading and trailing edge structure including flaps and ailerons.

### III. SHORTENED FORM OF WING WEIGHT EQUATION

The terms of Eq.(5) were evaluated for several wings of contemporary commercial and military airplanes. It was found that the terms representing rib flange material and the loading term for the rib shear material, Eqs.(3) and (1f), could be omitted with no appreciable effect on the final numerical result.

The nonoptimum factor  $k_x$  is defined as the ratio of the actual structural box weight to the weight of the optimum structural box. It was originally intended to determine values of  $k_x$  by comparing calculated values of Eq.(5) with actual values of weight for the airplanes studied. However, much of the weight data available at this time was not itemized in sufficient detail to evaluate  $J_{LT}$  with the required accuracy. Therefore, a new nonoptimum factor was defined and used in connection with the statistical study.

To avoid introducing any additional error by using uncertain estimates for values of  $J_{LT}$ , the new nonoptimum factor is defined so as to include the weight of secondary structure (leading and trailing edge structure).

If the assumption is made that the weight of the secondary structure is proportional to the weight of the primary structure, the wing weight is given by

$$W_w = (k_x + K_1) \times (\text{optimum weight of structural box}). \quad (6)$$

Let the new nonoptimum factor be defined as

$$k'_x = k_x + K_1. \quad (7)$$

Then the over-all weight becomes

$$W_w = k'_x \times (\text{optimum weight of structural box}). \quad (8)$$

In this form,  $k'_x$  could be readily evaluated and was found to be a function of planform taper ratio, as predicted in Report R-100. Due to the classified nature of the weight data involved, actual values are not included in this unclassified report.

Omitting the rib weight terms discussed previously, the shortened form of the wing weight equation is now written

$$W_w = k'_x w \left[ \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_h K_B}{K_L \bar{C}} \right) 144 \bar{C} S \right. \\ \left. + \frac{1}{2} \frac{J_n n_f W_g}{F_a} \left( \frac{1}{2} k_{is} \frac{J_s}{m_s} + \frac{1}{3} k_{ib} \frac{k_b}{k_e} \frac{(b/2)}{h_R} \right) b \right] \quad (9)$$

where  $k'_x = \frac{\text{actual wing weight}}{\text{optimum weight of structural box}} .$

#### IV. INERTIA LOAD RELIEF

As shown in Report R-100, part of the wing structural material is proportional to the size of the wing and part is proportional to the loading. The integration factors represent, in nondimensional form, the volume of material which is proportional to loading. These factors are calculated separately for shear material and bending material.

The value of an integration factor is determined only by the span-wise distribution of shear load or axial flange load. It was assumed in Report R-100 that the dead-weight loads had the same distribution as the airloads. Therefore, the distribution of net loads, when plotted nondimensionally, was the same as for airloads and consequently had the same value of integration factor. All that was necessary in order to include the effects of inertia loads was to correct the value of loads at the root section. This was done in Report R-100 by correcting the shear at the root by multiplying by the factor  $J_n$ , where

$$J_n = \frac{W_g - W_{wc}}{W_g} \quad (10)$$

and where  $W_g$  = airplane gross weight

$W_{wc}$  = weight of wing and contents.

(See Eq.10a, Report R-100.)

Since it is no longer assumed that the inertia loads are distributed in the same manner as the airloads, the simple correction factor shown above cannot be used. A study of modern airplanes has shown that the difference in values of airload integration factors and net load integration factors may be of considerable magnitude in some cases. Therefore, a method was derived for treating the airload and inertia-load integration factors separately and combining them later to obtain net values. The method is presented below.

The integration factors are obtained from integration of the loads expressed in nondimensional form. For shear, the factor is obtained from an integration of the shear loads. For bending material, it is obtained from an integration of the axial flange load expressed nondimensionally. Since the portion of the material considered here is operating



at constant allowable stresses, the integration factor also represents the amount of material necessary for carrying the shear or bending.

The airload integration factors have been described previously in Report R-100. The total inertia-load integration factors may be composed of several factors which are obtained separately for concentrated loads and for distributed loads and which are then combined. A procedure for the calculation of inertia-load integration factors is outlined in Section V.

After the airload and total inertia-load integration factors have been obtained, they are used to calculate values of the inertia relieving load factor  $J_n$ . This factor is determined separately for shear and bending material. As derived in Appendix A, the expressions for  $J_n$  are found to be as follows:

*Shear Material,*

$$J_{ns} = 1 - \frac{W_{wc}}{W_g} \frac{k_{i_{sd}}}{k_{i_{sa}}} \quad (11)$$

where  $k_{i_{sd}}$  = shear integration factor for inertia loads (total)

$k_{i_{sa}}$  = shear integration factor for airloads;

*Bending Material,*

$$J_{nb} = 1 - \frac{W_{wc}}{W_g} \frac{k_{bd}}{k_{ba}} \frac{k_{i_{bd}}}{k_{i_{ba}}} \quad (12)$$

where  $k_{bd}$  = span-wise load distribution factor ( $= k_{i_{sd}}$ ) for inertia loads

$k_{ba}$  = span-wise load distribution factor ( $= k_{i_{sa}}$ ) for airloads

$k_{i_{bd}}$  = bending integration factor for inertia loads

$k_{i_{ba}}$  = bending integration factor for airloads.

The span distribution factor  $k_b$  is identical with the shear integration factor  $k_i$ . Therefore, values calculated for  $k_i$ , may be used in Eq.(12).

The two terms  $J_{ns}$  and  $J_{nb}$  are substituted in the wing weight equation in place of the single value of  $J_n$  previously used.

The shear reduction factor  $J_s$  also required revision in view of the more general assumptions made regarding inertia

load distribution. The derivation of Appendix B gives the expression for the new shear reduction factor as

$$J'_s = 1 - \frac{k_{i_{ba}}(1-m)}{3k_e} \left[ \frac{J_{nb}}{J_{ns}} \right] \quad (13)$$

where  $m$  = thickness taper ratio =  $\frac{\text{tip thickness}}{\text{root thickness}}$   
 $k_e$  = effective depth factor.

The corrected wing weight equation is obtained by substituting the new values of  $J_n$  and  $J_s$  in Eq.(5) of Section II:

$$\begin{aligned} W_w = wk_x \left\{ \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_B K_h}{K_L \bar{C}} \right) 144 \bar{C} S \right. \\ \left. + \frac{1}{2} \frac{n_f W_g}{F_a} \left[ \left( \frac{1}{2} \frac{J_{ns} k_{i_{sa}} J'_s}{m_s} + \frac{1}{3} J_{nb} k_{i_{ba}} \frac{k_{ba}}{k_e} \frac{(b/2)}{h_R} \right) b \right. \right. \\ \left. \left. + K_B \bar{C} \left( \frac{K_B}{2K_h m_{ar}} + \frac{1}{m_s} \right) \right] \right\} + J_{LT} S. \quad (14) \end{aligned}$$

Substituting the new values of  $J_n$  and  $J_s$  in the shortened form of the wing weight equation, given by Eq.(9), yields the expression

$$\begin{aligned} W_w = k'_x w \left[ \left( 2C_s k_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_h K_B}{K_L \bar{C}} \right) 144 \bar{C} S \right. \\ \left. + \frac{1}{2} \frac{n_f W_g b}{F_a} \left( \frac{1}{2} \frac{k_{i_{sa}} J_{ns} J'_s}{m_s} + \frac{1}{3} k_{i_{ba}} J_{nb} \frac{k_{ba}}{k_e} \frac{(b/2)}{h_R} \right) \right]. \quad (15) \end{aligned}$$

In the case of a sweptback wing, the values of structural span and structural mean aerodynamic chord are used. The structural span  $b_s$  is defined as

$$b_s = \frac{b}{\cos \Lambda} \quad (16)$$

where  $\Lambda$  = sweepback angle measured at the quarter-chord position.

The structural mean aerodynamic chord  $\bar{C}_s$  is defined as

$$\bar{C}_s = \bar{C} \cos \Lambda. \quad (17)$$

Equation (17) gives exact values only for nontapered wings. However, except for low-aspect-ratio highly tapered designs, these expressions give sufficient accuracy when used with tapered wings.

In order to write Eq.(15) in a more general form, including the effects of sweepback, let

$$\beta' = \left( 2C_s K_s^2 K_h^2 + C_c k_L K_L K_B + \frac{\bar{t}_{r0} K_h K_B}{K_L \bar{C}_s} \right) \quad (18)$$

$$\theta' = \left( \frac{1}{2} \frac{k_{i_{sa}} J_{ns} J'_s}{m_s} + \frac{1}{3} k_{i_{ba}} J_{nb} \frac{k_{ba}}{k_e} \frac{(b_s/2)}{h_R} \right) . \quad (19)$$

Substituting in Eq.(15), the wing weight becomes

$$W_w = k'_x w \left( 144 \beta' \bar{C}_s S + \frac{\theta' b_s W_g n_f}{2 F_a} \right) . \quad (20)$$

## V. METHODS FOR ESTIMATING INERTIA-LOAD INTEGRATION FACTORS

For estimating the integration factors, the inertia loads are broken down into two general types—distributed and concentrated. The distributed loads come from the wing structure and from miscellaneous distributed items such as control systems. The concentrated loads originate with wing-mounted engines, with landing gear, and sometimes with fuel tanks. It is usually more convenient to treat the fuel tanks as several concentrated loads rather than as a distributed load, since a restricting assumption is made regarding the manner in which the distributed loads vary.

Integration factors are calculated separately for the different loads and are then combined into a single factor. These factors are calculated separately for shear loads and for bending loads.

### CONCENTRATED LOADS

The expression for the shear integration factor is derived below for the case of several concentrated loads located at different span-wise stations. This loading condition is shown in Fig.1.

The shear curve for this loading is shown in Fig.2.

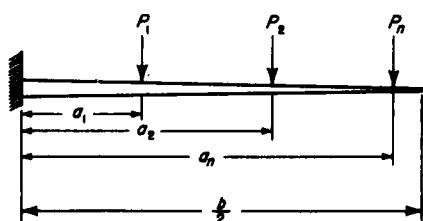


Fig. 1—Concentrated loading condition

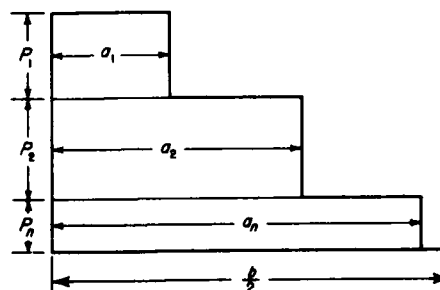


Fig. 2—Shear diagram for concentrated loads

The area under the shear curve of Fig.2 is given by

$$\text{Area} = P_1 a_1 + P_2 a_2 + P_n a_n . \quad (21)$$

This may be expressed in nondimensional form by dividing by the value of shear at the root and by the semispan (see Appendix F, Report R-100):

$$\text{Area}' = \frac{P_1 a_1 + P_2 a_2 + P_n a_n}{(P_1 + P_2 + P_n)(b/2)} \quad (22)$$

Since the allowable shear stress is a constant for this portion of the material, Eq.(22) also represents the volume of shear material expressed in nondimensional form.

The integration factor for shear is defined as the ratio of the area under the shear curve to that under the shear curve for the basic case. Since the area equals one-half\* for the basic case, the shear integration factor for concentrated loads is given by

$$k_{i,s} = \frac{2}{(b/2)} \frac{P_1 a_1 + P_2 a_2 + P_n a_n}{P_1 + P_2 + P_n} \quad (23)$$

The integration factor for bending depends on the distribution of the axial flange load  $P_a$ , where

$$P_a = \frac{M}{h} \quad (24)$$

and where  $M$  = bending moment

$h$  = depth of the wing.

The integration factor  $k_{i,b}$  for bending is defined as follows:

Let  $A_0$  = area under the curve of  $M/h$  versus distance along the semispan for the given loading condition

$A_b$  = area under the curve of  $M/h$  versus distance along the semispan for the basic case.

Then

$$k_{i,b} = \frac{A_0}{A_b} \quad (25)$$

(see Report R-100, page 47.)

For convenience, these areas are expressed in nondimensional form. Since the area (nondimensional) under the curve of  $M/h$  for the basic case equals one-third,\* the bending integration factor is written

$$k_{i,b} = 3A'_0 \quad (26)$$

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\* See RAND Report R-100, pp.46 and 47.

where

$$A'_0 = \frac{1}{\frac{M_R}{h_R} \frac{b}{2}} \int_0^{b/2} \frac{M}{h} dx . \quad (27)$$

For the general case of a tapered wing under a concentrated load, the integration factor is given by

$$k_{ibd} = \left( \frac{3}{1-\phi} \right) \left( \frac{1}{1-m} \right)^2 \{ 1 - [m + \phi(1-m)][1 - \log_e(m + \phi(1-m))] \} \quad (28)$$

where  $\phi = 1 - \eta$ , where  $\eta$  is the nondimensional distance from the root to the span-wise station at which the concentrated load is located

$$m = \frac{\text{tip thickness}}{\text{root thickness}} .$$

A plot of Eq. (28) is shown in Fig. 3. A value of  $k_{ib}$  is read from the curves for each concentrated load. These separate values are then combined into a single integration factor by the expression

$$k_{ibd} = \frac{k_{ib1} P_1 \eta_{P1} + k_{ib2} P_2 \eta_{P2} + \dots + k_{ibn} P_n \eta_{Pn}}{P_1 \eta_{P1} + P_2 \eta_{P2} + \dots + P_n \eta_{Pn}} \quad (29)$$

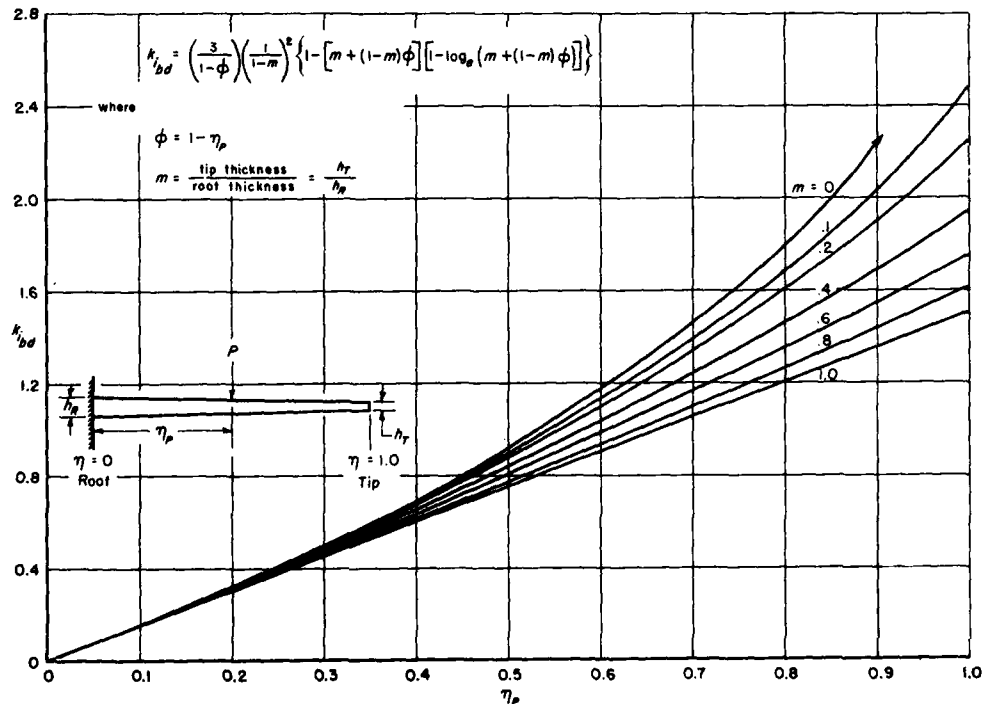


Fig. 3—Integration factors for bending material—concentrated loads

where  $k_{ib}$  = value read from the curves of Fig.3

$P$  = value of concentrated load

$\eta_p$  = nondimensional distance from the location of the load to the wing root section.

Equation (29) is a special case of Eq.(33), which is derived in Appendix D.

### DISTRIBUTED LOADS

The integration factors for distributed loads are derived on the basis of the following assumption—that the load at any span-wise station is proportional to the cross-sectional area of the wing at that station.\* Such a load distribution would result from the exact filling of the enclosed volume of the wing by a material of constant density.

The shear at any station is proportional to the enclosed wing volume outboard of that station. An algebraic solution for the shear integration factor yields the expression

$$k_{isd} = \frac{1 + \lambda + d\lambda + 3d\lambda^2}{2 + \lambda + d\lambda + 2d\lambda^2} \quad (30)$$

where  $d = m/\lambda$ . A plot of this function is shown in Fig.4.

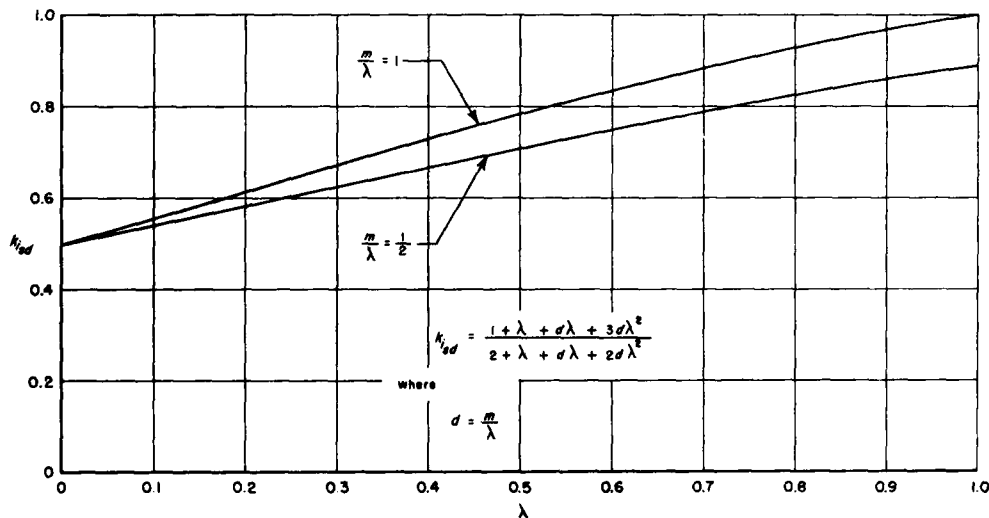


Fig. 4—Shear integration factor for weight distributed as wing volume

\* Reference 3 has shown that the span-wise distribution of structure weight for tapered wings tended to vary as the square of the chord. For a wing having a constant airfoil section along the span, such a distribution would also vary as the wing cross-sectional area.

Similarly, the solution for the bending integration factor is given by

$$k_{ibd} = B \left[ \frac{a\lambda}{2} \left( \frac{1}{2} - a + a^2 \log_e \frac{1+a}{a} \right) + \frac{m-2m\lambda-2}{6(1-m)} \left( \frac{1}{3} - \frac{a}{2} + a^2 \right. \right. \\ \left. \left. - a^3 \log_e \frac{1+a}{a} \right) + \frac{1-\lambda}{12} \left( \frac{1}{4} - \frac{a}{3} + \frac{a^2}{2} - a^3 + a^4 \log_e \frac{1+a}{a} \right) \right] \quad (31)$$

where

$$B = \frac{3}{\frac{m\lambda}{2} + \frac{m-2m\lambda+\lambda}{6} + \frac{(1-\lambda)(1-m)}{12}}$$

$$a = \frac{m}{1-m}$$

This expression is shown graphically in Fig.5.

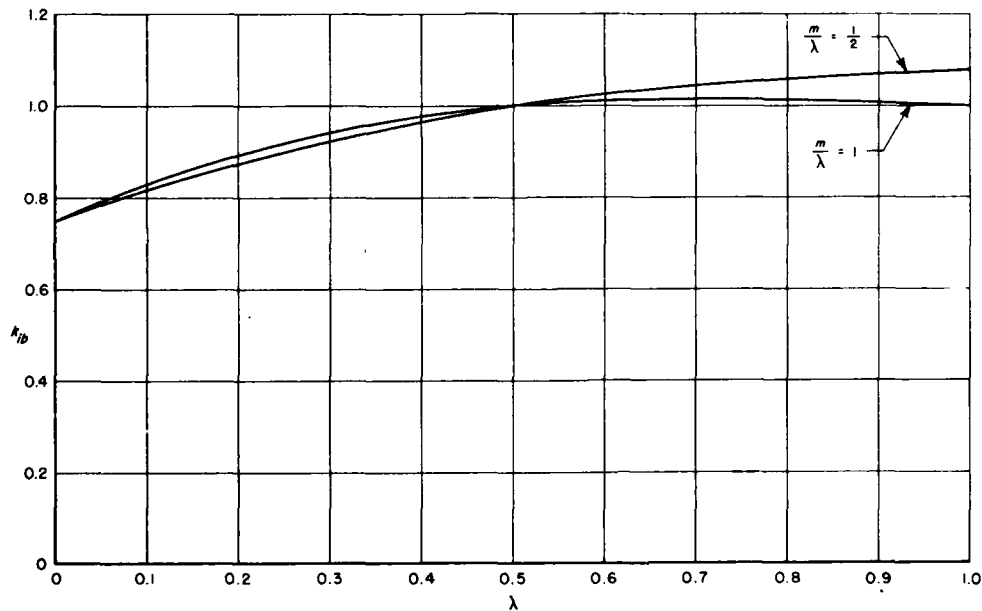


Fig. 5—Integration factors for bending material for inertia loads distributed as wing volume

If it becomes desirable to revise the assumptions regarding the variation of distributed inertia loads in the wing, integration factors can be derived for any assumed variation of distributed loads. These factors would then be used in the same manner as those derived here.



### TOTAL COMBINED VALUES

After values of the integration factors have been calculated separately for concentrated and distributed loads, these values must be combined to give total values for inertia-load integration factors. This is done for the shear and bending factors by means of the expressions given below.

The total value for the inertia-load shear integration factor is given by

$$k_{isd} = \frac{k_{is1} V_{R1} + k_{is2} V_{R2} + \dots + k_{isn} V_{Rn}}{V_{R1} + V_{R2} + \dots + V_{Rn}} \quad (32)$$

where  $k_{is}$  = shear integration factor for any given load.

$V_R$  = corresponding value of shear at the root.

The total integration factor for bending material is given by

$$k_{ibd} = \frac{k_{ib1} M_{R1} + k_{ib2} M_{R2} + \dots + k_{ibn} M_{Rn}}{M_{R1} + M_{R2} + \dots + M_{Rn}} \quad (33)$$

where  $k_{ib}$  = bending material integration factor for any given load

$M_R$  = corresponding value of bending moment at the root.

The derivation of these expressions is given in Appendix D.

## VI. CONCLUSIONS

In order to improve the accuracy of the equation for estimating wing weight it was necessary to use a more accurate method of estimating the loads on the wing structure. This required a more precise procedure for estimating the effects of dead-weight loads, such as the method outlined in this report. The use of this refined method is more important for large airplanes having wing fuel and wing-mounted engines than for airplanes which have no sources of large concentrated loads in the wings.

The shortened form of the weight equation (Eq.20) using the over-all nonoptimum factor  $k'_x$  is the most convenient form to use in the estimation of wing weights on the basis of currently available weight data. The nonoptimum factor  $k'_x$  may be determined from statistics and expressed as a function of planform taper ratio.

Values of airload and inertia-load integration factors were obtained from actual bending moment curves and were checked against values calculated by the methods outlined in this report. The values were in good agreement, the best correlation being obtained for the larger airplanes.

## APPENDIX A

### DERIVATION OF EXPRESSIONS FOR INERTIA-LOAD RELIEF FACTORS

In the derivation of integration factors for the shear and bending materials (see Report R-100) it was assumed that the dead weight of wing and contents had the same distribution as the airloads. The use of  $J_n$  as outlined previously gives the correct net values of shear and bending moment at the wing root section but does not account for the actual distribution of dead weight.

An investigation of the distribution of dead-weight shear and bending moments for actual airplanes has shown that the use of  $J_n$  alone yields somewhat optimistic values for the weights of shear and bending material. This is particularly true for large airplanes having engines mounted in the wings.

In order to make the wing weight equations more accurate by accounting for the actual distribution of dead weight, the following derivation is presented.

The shear and bending materials discussed here are those portions only which vary with loading and which operate at constant allowable stresses. The portions which are proportional to size are not affected.

#### SHEAR MATERIAL

Equation (84) of Report R-100 gives the expression for the volume of shear material as

$$(\text{Vol})_s = \frac{1}{2} k_{is} A_{sR} b \quad (34)$$

where  $k_{is}$  = integration factor derived on the basis of airloads

$A_{sR}$  = area of shear material required at the root section and given by

$$A_{sR} = \frac{1}{2} \frac{W_g n_f J_s J_n}{F_{s0}} \quad (35)$$

Substituting Eq.(35) in Eq.(84), the volume of shear material becomes

$$(\text{Vol})_s = \frac{W_g n_f J_s b k_{is} J_n}{4 F_{s0}} \quad (36)$$

The use of  $J_n$  gives the required area of shear material corrected for the relieving shear from dead weight. However, in actual airplane wings, the value of shear relief given by the term  $J_n$  is correct only at the root section, since the dead-weight shear is not proportional to the airload shear at all span-wise stations. This difference can be accounted for by using a combination of two integration factors—one for airloads and one for the relieving dead-weight loads.

The net cross-sectional area of shear material required at any station is given by

$$A_s = \frac{V_a - V_d}{F_{s_0}} \quad (37)$$

where  $V_a$  = shear from airloads

$V_d$  = shear from dead-weight loads.

The volume of shear material is found by integration of Eq.(37):

$$(\text{Vol})_s = \frac{b}{F_{s_0}} \int_0^1 V_a d\eta - \frac{b}{F_{s_0}} \int_0^1 V_d d\eta. \quad (38)$$

The subscript  $R$  designates values taken at the root section, and the value of  $V_a$  at the root section is given by

$$V_{aR} = \frac{1}{2} W_g n_f J_s. \quad (39)$$

The value of  $V_d$  at the root section is given by

$$V_{dR} = \frac{1}{2} W_{wc} n_f J_s \quad (40)$$

where  $W_{wc}$  = weight of wing and contents.

Multiplying and dividing the first and second terms of Eq.(38) by Eqs.(39) and (40), respectively, the volume of shear material may be written

$$(\text{Vol})_s = \frac{W_g n_f J_s b}{2F_{s_0}} \int_0^1 \frac{V_a}{V_{aR}} d\eta - \frac{W_{wc} n_f J_s b}{2F_{s_0}} \int_0^1 \frac{V_d}{V_{dR}} d\eta. \quad (41)$$

This assumes that the shear from airloads is greater than the dead-weight shear at all span-wise stations and was true for the most critical design condition for all of the airplanes studied.

Since the allowable shear stress is constant, the cross-sectional area of shear material is directly proportional to the shear load. Therefore,

$$\frac{V_a}{V_{aR}} = \frac{A_{s_a}}{A_{s_{aR}}} \quad (42)$$

where  $A_{s_a}$  = area of shear material which would be required to carry airloads only. Also,

$$\frac{V_d}{V_{dR}} = \frac{A_{s_d}}{A_{s_{dR}}} \quad (43)$$

where  $A_{s_d}$  = area of shear material which would be required to carry dead-weight loads only. Substituting Eqs.(42) and (43) into Eq.(41) and factoring,

$$(Vol)_s = \frac{n_f J_s b}{2F_{s_0}} \left[ W_g \int_0^1 \frac{A_{s_a}}{A_{s_{aR}}} d\eta - W_{wc} \int_0^1 \frac{A_{s_d}}{A_{s_{dR}}} d\eta \right]. \quad (44)$$

Since the integration factors are evaluated here for both airloads and inertia loads, it is necessary to use subscripts  $a$  (airload) and  $d$  (dead-weight load) with the integration factors. The value of  $k_{i_{sa}}$  of this report corresponds to  $k_{i_s}$  of Report R-100,  $k_{i_{ba}}$  corresponds to  $k_{i_b}$ , etc.

The integration factor for airloads is defined as

$$k_{i_{sa}} = 2 \int_0^1 \frac{A_{s_a}}{A_{s_{aR}}} d\eta. \quad (45)$$

This expression is the same as that given by Eq.(F-1) in Report R-100.

The integration factor for dead-weight loads is defined as

$$k_{i_{sd}} = 2 \int_0^1 \frac{A_{s_d}}{A_{s_{dR}}} d\eta. \quad (46)$$

This factor can be evaluated from a knowledge of the distribution of the dead weight of wing and contents.

Substituting Eqs.(45) and (46) into Eq.(44), the volume of shear material becomes

$$(Vol)_s = \frac{n_f J_s b}{4F_{s_0}} (W_g k_{i_{sa}} - W_{wc} k_{i_{sd}}). \quad (47)$$

Factoring out the terms associated with airloads,

$$(\text{Vol})_s = \frac{W_g n_f J_s b k_{i_{sa}}}{4F_{s_0}} \left( 1 - \frac{W_{wc}}{W_g} \frac{k_{i_{sd}}}{k_{i_{sa}}} \right). \quad (48)$$

Equation (48) is the same as Eq.(36) with  $J_n$  replaced by the term in parenthesis. It can be seen that this term, which represents an integrated dead-weight relief factor, reduces to  $J_n (= 1 - W_{wc}/W_g)$  when the integration factors are equal. This would occur if, as previously assumed, the dead weight were distributed in the same manner as the airloads. The revised inertia-load integration factor for shear material will be denoted by  $J_{n_s}$ , where

$$J_{n_s} = 1 - \frac{W_{wc}}{W_g} \frac{k_{i_{sd}}}{k_{i_{sa}}}.$$

#### BENDING MATERIAL

The volume of bending material which is proportional to loading is given by Eq.(85) of Report R-100:

$$(\text{Vol})_b = \frac{1}{3} k_{i_b} A_{b_R} b \quad (49)$$

where  $k_{i_b}$  = integration factor calculated on the basis of airloads

$A_{b_R}$  = cross-sectional area of bending material required at the root section, and where

$$A_{b_R} = \frac{W_g n_f}{2} \left( \frac{b/2}{h_R} \right) \frac{k_b J_n}{k_e F_a}. \quad (50)$$

Substituting Eq.(50) into Eq.(49), the volume of bending material is written

$$(\text{Vol})_b = \frac{W_g n_f}{6} \left( \frac{b/2}{h_R} \right) \frac{b k_b k_{i_b} J_n}{k_e F_a}. \quad (51)$$

Here, as in Eq.(36),  $J_n$  gives the correct value for dead-weight load relief effects only at the root section. The span-wise load distribution factor  $k_b$  and the integration factor  $k_{i_b}$  for bending material are based only on airloads.

The net cross-sectional area of bending material required at any station is given by

$$A_b = \frac{2}{F_a k_e h} (M_a - M_d) \quad (52)$$

where  $M_a$  = bending moment from airloads

$M_d$  = bending moment from dead-weight loads.

The volume of bending material is found by integrating Eq. (52):

$$(Vol)_b = \frac{2b}{F_a k_e} \int_0^1 \frac{M_a}{h} d\eta - \frac{2b}{F_a k_e} \int_0^1 \frac{M_d}{h} d\eta. \quad (53)$$

This assumes that the bending moments from airloads are greater than those from dead-weight loads at all span-wise stations.

The value of  $M_a/h$  at the root section is given by

$$\frac{M_{aR}}{h_R} = \frac{1}{4} W_g n_f \left( \frac{b/2}{h_R} \right) k_{b_a} \quad (54)$$

where  $k_{b_a}$  = span-wise load distribution factor for airloads.

The value of  $M_d/h$  at the root section is given by

$$\frac{M_{dR}}{h_R} = \frac{1}{4} W_{wc} n_f \left( \frac{b/2}{h_R} \right) k_{b_d} \quad (55)$$

where  $k_{b_d}$  = span-wise load distribution factor for dead-weight loads.

Multiplying and dividing the first and second terms of Eq. (53) by Eqs. (54) and (55), respectively, the volume of bending material is written

$$(Vol)_b = \frac{W_g n_f \left( \frac{b/2}{h_R} \right) b k_{b_a}}{2F_a k_e} \int_0^1 \frac{M_a/h}{M_{aR}/h_R} d\eta - \frac{W_{wc} n_f \left( \frac{b/2}{h_R} \right) b k_{b_d}}{2F_a k_e} \int_0^1 \frac{M_d/h}{M_{dR}/h_R} d\eta. \quad (56)$$

Since the allowable axial stress is constant, the cross-sectional area of bending material required at any station is directly proportional to the quantity  $M/h$ . Therefore,

$$\frac{A_{b_a}}{A_{b_{aR}}} = \frac{M_a/h}{M_{aR}/h_R} \quad (57)$$

where  $A_{b_a}$  = area of bending material which would be required to carry airloads alone. Also,

$$\frac{A_{b_d}}{A_{b_{dR}}} = \frac{M_d/h}{M_{dR}/h_R} \quad (58)$$

where  $A_{b_d}$  = cross-sectional area of bending material which would be required to carry only dead-weight loads.

Substituting Eqs.(57) and (58) into Eq.(56) and factoring, the volume of bending material becomes

$$(\text{Vol})_b = \frac{1}{2} \left( \frac{b/2}{h_R} \right) \frac{bn_f}{F_a k_e} \left[ W_g k_{b_a} \int_0^1 \frac{A_{b_a}}{A_{b_{aR}}} d\eta - W_{wc} k_{b_d} \int_0^1 \frac{A_{b_d}}{A_{b_{dR}}} d\eta \right]. \quad (59)$$

The integration factor for bending material required by airloads is defined as

$$k_{i_{b_a}} = 3 \int_0^1 \frac{A_{b_a}}{A_{b_{aR}}} d\eta. \quad (60)$$

This expression is the same as the original equation for the bending-material integration factor given by Eq.(F-6) of Appendix F, Report R-100.

The integration factor for bending material which would be required to carry only dead-weight loads is defined as

$$k_{i_{b_d}} = 3 \int_0^1 \frac{A_{b_d}}{A_{b_{dR}}} d\eta. \quad (61)$$

Substituting Eqs.(60) and (61) into Eq.(59), the volume of bending material is written

$$(\text{Vol})_b = \frac{1}{6} \left( \frac{b/2}{h_R} \right) \frac{bn_f}{F_a k_e} (W_g k_{b_a} k_{i_{b_a}} - W_{wc} k_{b_d} k_{i_{b_d}}). \quad (62)$$



Factoring out the quantities associated only with airloads, the volume of bending material becomes

$$(Vol)_b = \frac{W_g n_f}{6} \left( \frac{b/2}{h_R} \right) \frac{b k_{ba} k_{iba}}{F_a k_e} \left( 1 - \frac{W_{wc}}{W_g} \frac{k_{bd}}{k_{ba}} \frac{k_{ibd}}{k_{iba}} \right). \quad (63)$$

This expression for volume of bending material is the same as Eq.(51), except that the dead-weight relief factor  $J_n$  is replaced by the integrated relief factor in parenthesis. This revised inertia-load relief factor for bending material is denoted by  $J_{nb}$ , where

$$J_{nb} = 1 - \frac{W_{wc}}{W_g} \frac{k_{bd}}{k_{ba}} \frac{k_{ibd}}{k_{iba}}.$$

It can be shown algebraically that the integration factor  $k_{i_s}$  for shear material is identical with the span-wise load distribution factor  $k_b$ . For purposes of explanation the two factors are left in their original form, but this identity should be kept in mind when calculating actual values.

Some actual values for the integration factors and relief factors are given below:

Item	Airplane	
	A	B
1. $k_{isa}$	.810	.806
2. $k_{isd}$	.599	.565
3. $k_{iba}$	1.128	1.030
4. $k_{ibd}$	.706	.813
5. $J_n$	.546	.647
6. $J_{na}$	.665	.752
7. $J_{nb}$	.790	.805
8. Percent increase in weight of shear material by use of corrected relief factor	22	16
9. Percent increase in weight of bending material by use of corrected relief factor	45	24

Designation of airplanes:

A = large four-engine cargo airplane calculated for the critical condition of no wing fuel

B = fighter aircraft with no engines in the wings calculated for the critical condition of no wing fuel.

The errors introduced by using  $J_n$  rather than the integrated relief factor can be seen by comparing Items 6 and 7 with Item 5. The percentage error is seen to be larger for the four-engine cargo airplane.

When more detailed data are acquired on wing loads for actual airplanes, these factors ( $J_n$ , and  $J_{n_i}$ ) can be evaluated for a number of airplanes, and perhaps average values can be computed for different classes of airplanes.

## APPENDIX B

### DERIVATION OF THE INTEGRATED SHEAR REDUCTION FACTOR

The expression for the shear reduction factor  $J_s$  given by Eq.(A-8) of Report R-100 is valid only at the root section of the wing. An investigation of actual airplane wings has shown that the variation in the value of  $J_s$  along the span makes necessary the use of an average or "integrated" value. This integrated shear reduction factor, when substituted in the general wing weight equation, yields an amount of shear material which is corrected for the over-all effect of shear relief along the span.

The integrated shear reduction factor, denoted by  $J'_s$ , is defined as follows:

$$J'_s = \frac{(\text{Vol})_{s_c}}{(\text{Vol})_{s_0}} \quad (64)$$

where  $(\text{Vol})_{s_c}$  = volume of shear material necessary to carry the shear load after it is corrected for relieving shear loads from the flanges

$(\text{Vol})_{s_0}$  = volume of shear material necessary to carry the total uncorrected shear load.

This expression may also be written

$$J'_s = 1 - \frac{(\text{Vol})_{s_f}}{(\text{Vol})_{s_0}} \quad (65)$$

where  $(\text{Vol})_{s_f}$  = volume of material which is subtracted from  $s_0$  when relieving shear is considered.

The volume of shear material before correcting for shear relief is

$$(\text{Vol})_{s_0} = \frac{1}{2} \frac{V_R k_{is} b}{F_{s_0}} \quad (66)$$

where  $V_R$  = maximum shear load at the root of one wing.

The volume of shear material "removed" by considering shear relieving loads from the beam flanges is

$$(\text{Vol})_{sf} = 2 \int_0^{b/2} \frac{V_f}{F_{s0}} dy \quad (67)$$

where  $V_f$  = portion of the shear load resisted by axial loads in the flanges. The shear  $V_f$  is given by Eq.(A-2) of Report R-100 as

$$V_f = P \tan \theta . \quad (68)$$

For the general case, the rate of change of depth per unit span is given by

$$\tan \theta = \frac{h_R (1-m)}{b/2} \quad (69)$$

where  $m = \frac{\text{depth at tip}}{\text{depth at root}} .$

The flange load  $P$  is given by

$$P = \frac{M}{k_e h} . \quad (70)$$

The depth  $h$  at any station  $y$  is given by

$$h = h_R \left[ 1 - \frac{y}{b/2} (1-m) \right] . \quad (71)$$

Substituting Eqs.(71), (70), and (69) into Eq.(68), the portion of the shear load resisted by the flanges is expressed as

$$V_f = \frac{M h_R (1-m)}{k_e \left( \frac{b}{2} \right) h_R \left[ 1 - \frac{y}{b/2} (1-m) \right]} \quad (72)$$

Simplifying,

$$V_f = \frac{M (1-m)}{k_e \left( \frac{b}{2} \right) \left[ 1 - \frac{y}{b/2} (1-m) \right]} . \quad (73)$$

Substituting Eq.(73) into Eq.(67) and expressing the span-wise distance  $y$  in terms of  $\eta$ , where  $\eta = y/(b/2)$ ,

$$(\text{Vol})_{sf} = \frac{2(1-m)}{k_e F_{s_0}} \int_0^1 \frac{M}{1-\eta+\eta m} d\eta. \quad (74)$$

Multiplying and dividing by the bending moment  $M_R$  at the root section,

$$(\text{Vol})_{sf} = \frac{2(1-m)M_R}{k_e F_{s_0}} \int_0^1 \frac{M/M_R}{1-\eta+\eta m} d\eta. \quad (75)$$

Equations (F-5) and (F-6) of Appendix F, Report R-100, show that

$$\int_0^1 \frac{M/M_R}{1-\eta+\eta m} d\eta = \frac{k_{ib}}{3} \quad (76)$$

where  $k_{ib}$  = integration factor for bending material.

Substituting Eq.(76) into Eq.(75),

$$(\text{Vol})_{sf} = \frac{2}{3} \frac{(1-m)M_R k_{ib}}{k_e F_{s_0}}. \quad (77)$$

Substituting Eqs.(77) and (66) into Eq.(65), the integrated shear reduction factor becomes

$$J'_s = 1 - \frac{4}{3} \left( \frac{M_R}{V_R} \right) \frac{k_{ib}(1-m)}{k_e k_{is} b}. \quad (78)$$

As shown in Appendix F of Report R-100, the following expression may be written

$$\frac{M_R}{V_R} = k_b \left( \frac{b}{4} \right) = k_{is} \left( \frac{b}{4} \right). \quad (79)$$

Substituting Eq.(79) into Eq.(78) and simplifying, the integrated shear reduction factor is written

$$J'_s = 1 - \frac{k_{ib}(1-m)}{3k_e}. \quad (80)$$

The expression for  $J'_s$  given by Eq.(80) is derived here to replace Eq.(A-8) of Report R-100. Equation (80) has been derived on the basis of net shear and bending loads. When the airloads and inertia loads are treated separately, it is necessary to go through the derivation above and to express the net moment  $M$  as  $M_a - M_d$ , the shear  $V$  as  $V_a - V_d$ ,

etc. (The subscript *a* denotes airload values and *d* denotes dead-weight values.) Such a derivation gives the expression for  $J'_s$  as

$$J'_s = 1 - \frac{k_{iba}(1-m)}{3k_e} \left[ \frac{J_{nb}}{J_{ns}} \right] . \quad (81)$$

## APPENDIX C

### DERIVATION OF REVISED RIB WEIGHT EXPRESSIONS

The original expressions for the volume of rib material are given in Section II as Eqs.(1e) through (1g). The derivations of these expressions are given in Report R-100. The derivation for the new rib weight terms is given below.

#### RIB SHEAR MATERIAL

The expression for the area (average volume per foot of span) of rib shear material is given by Eq.(47) of Report R-100 as

$$A'_{rs} = \frac{C_s h^2 B}{L} + \frac{1}{2} \frac{n_f J_n W_g BC}{(144S)F_{s_0}}. \quad (82)$$

Since the term  $C_s$  is the intercept (at  $V/h^2 = 0$ ) value of  $\bar{t}/h$  (see Fig.5, Report R-100), it represents a minimum allowable ratio of average thickness to depth. A study of actual airplanes showed that it is desirable to place the minimum-value limitation on thickness rather than on proportion.

Since  $C_s = (\bar{t}/h)_0$ , the minimum average thickness  $\bar{t}_{r_0}$  for the rib shear material is given by

$$\bar{t}_{r_0} = C_s h. \quad (83)$$

Substituting Eq.(83) into the first term of Eq.(82), the expression for the average area (volume per unit length of span) of rib shear material becomes

$$A'_{rs} = \bar{t}_{r_0} \frac{hB}{L} + \frac{1}{2} \frac{n_f J_n W_g BC}{(144S)F_{s_0}}. \quad (84)$$

The volume is found by integrating Eq.(84) over the span, as in Report R-100:

$$(\text{Vol})_{rs} = \frac{\bar{t}_{r_0} K_h K_B (144)S}{K_L} + \frac{1}{2} \frac{J_n n_f W_g K_B \bar{C}}{F_{s_0}}. \quad (85)$$

## RIB FLANGE MATERIAL

As previously stated, the new expression for the volume of rib flange material is derived on the basis of axial loads associated with chordwise bending moments.

From Fig.10 and Eq.(45) of Report R-100, it can be seen that the expression for the maximum chordwise bending moment is given by

$$M_{\max} = \frac{1}{8} n_f J_n \frac{W_g}{144S} LCB. \quad (86)$$

The axial load  $P$  in the rib flanges is given by

$$P = \frac{M}{h} = \frac{n_f J_n W_g LCB}{8h(144S)}. \quad (87)$$

The cross-sectional area, per bay, of rib flange material is

$$A = \frac{2P}{F_y} = \frac{n_f J_n W_g LCB}{4h(144S)F_y} \quad (88)$$

where  $F_y$  = compressive yield stress, or equivalent allowable stress.

The volume of rib flange material per bay is

$$\frac{(\text{Vol})_{rf}}{\text{bay}} = AB = \frac{n_f J_n W_g LCB^2}{4h(144S)F_y}. \quad (89)$$

Expressing  $L$ ,  $B$ , and  $h$  as fractions of chord, the average area (average volume per unit length of span) is written

$$A'_{rf} = \frac{n_f J_n W_g K_B^2 C^2}{4K_h(144S)F_y}. \quad (90)$$

Integrating Eq.(90) over the span gives the total volume of rib flange material as

$$(\text{Vol})_{rf} = \frac{n_f J_n W_g K_B^2 \bar{C}}{4K_h F_y}. \quad (91)$$



## APPENDIX D

### DERIVATION OF EXPRESSIONS FOR COMBINING INTEGRATION FACTORS

Since integration factors are calculated separately for various concentrated and distributed loads, they must be combined into single factors in order to be used in the wing weight equations. This may be done by means of the simple expressions derived below.

#### SHEAR INTEGRATION FACTORS

The shear integration factor represents, in nondimensional form, the area under the shear curve. Figure 6 shows the shear curves for two sources of load, as well as the total shear.

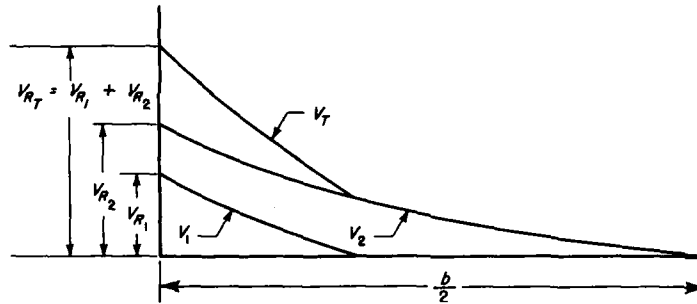


Fig. 6—Shear diagram for two superimposed  
loading conditions

The integration factor corresponding to shear curve  $V_1$  is given by

$$k_{i,1} = \frac{A_1}{\frac{1}{2} V_{R_1} \left( \frac{b}{2} \right)} \quad (92)$$

where  $A_1$  = area under curve of  $V_1$ .

Similarly, the integration factor for  $V_2$  is written

$$k_{i,2} = \frac{A_2}{\frac{1}{2} V_{R_2} \left( \frac{b}{2} \right)} \quad (93)$$

where  $A_2$  = area under curve of  $V_2$ .

The over-all integration factor  $k_{i,T}$ , which is to be written in terms of  $k_{i,1}$  and  $k_{i,2}$ , is given by

$$k_{i,T} = \frac{A_1 + A_2}{\frac{1}{2}(V_{R_1} + V_{R_2})\left(\frac{b}{2}\right)} \quad (94)$$

Solving Eqs.(92) and (93) for  $A_1$  and  $A_2$ , respectively, and substituting into Eq.(94), the total integration factor becomes

$$k_{i,T} = \frac{\frac{1}{2} k_{i,1} V_{R_1} \left(\frac{b}{2}\right) + \frac{1}{2} k_{i,2} V_{R_2} \left(\frac{b}{2}\right)}{\frac{1}{2}(V_{R_1} + V_{R_2})\left(\frac{b}{2}\right)} \quad (95)$$

Simplifying, the equation may be written in general form as

$$k_{i,T} = \frac{k_{i,1} V_{R_1} + k_{i,2} V_{R_2} + \dots + k_{i,n} V_{R_n}}{V_{R_1} + V_{R_2} + \dots + V_{R_n}} \quad (96)$$

This derivation assumes that the total shear does not change sign at any point along the span.

#### BENDING INTEGRATION FACTORS

The bending integration factor represents, in non-dimensional form, the area under the curve of  $M/h$ , which is the flange load. Figure 7 shows curves of  $M/h$  for two sources of bending loads and a curve for the total value of  $M/h$ .

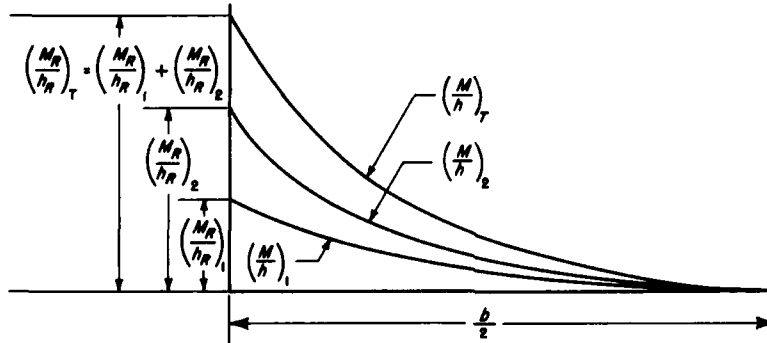


Fig. 7—Diagram of axial flange loads for two sources of bending moments

The bending integration factor for the curve of  $(M/h)_1$  is

$$k_{ib1} = \frac{A_1}{\frac{1}{3} \left( \frac{M_R}{h_R} \right)_1 \left( \frac{b}{2} \right)} \quad (97)$$

where  $A_1$  = area under curve of  $(M_R/h_R)_1$ .

Similarly, the integration factor for  $(M_R/h_R)_2$  is

$$k_{ib2} = \frac{A_2}{\frac{1}{3} \left( \frac{M_R}{h_R} \right)_2 \left( \frac{b}{2} \right)} \quad (98)$$

where  $A_2$  = area under curve of  $(M/h)_2$ .

The total integration factor is written

$$k_{ibT} = \frac{A_1 + A_2}{\frac{1}{3} \left[ \left( \frac{M_R}{h_R} \right)_1 + \left( \frac{M_R}{h_R} \right)_2 \right] \left( \frac{b}{2} \right)} \quad (99)$$

Solving Eqs.(97) and (98) for  $A_1$  and  $A_2$ , respectively, and substituting into Eq.(99), the total integration factor becomes

$$k_{ibT} = \frac{\frac{1}{3} k_{ib1} \left( \frac{M_R}{h_R} \right)_1 \left( \frac{b}{2} \right) + \frac{1}{3} k_{ib2} \left( \frac{M_R}{h_R} \right)_2 \left( \frac{b}{2} \right)}{\frac{1}{3} \left[ \left( \frac{M_R}{h_R} \right)_1 + \left( \frac{M_R}{h_R} \right)_2 \right] \left( \frac{b}{2} \right)} \quad (100)$$

Dividing out the wing root thickness  $h_R$  and simplifying, the total integration factor may be written in general as

$$k_{ibT} = \frac{k_{ib1} M_{R1} + k_{ib2} M_{R2} + \dots + k_{ibn} M_{Rn}}{M_{R1} + M_{R2} + \dots + M_{Rn}} \quad (101)$$

This derivation assumes that the total bending moment does not change sign at any point along the span.

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